

This finishes up the ancient part of our look at Geometry. Many people like to hark back to Euclid's axioms when talking geometry but I think it's MUCH better to go with modern axioms and be totally complete with our ideas and statements.

★ Video 2 for Chapter 1 Section 1.

Mostly new, more modern information. Popper 1.1 continues.

From 1958 – 1977 the Schoolhouse Mathematics Study Group met regularly to work on producing a study guide for a national curriculum at the high school level for Euclidean Geometry. They published the following in 1961.

The SMSG Axioms for Euclidean Geometry

Undefined terms

Axioms

Theorems and Definitions came along later.

SMSG Undefined Terms for Euclidean Geometry:

point, line, half plane, plane, and space

We take these as our beginning point. We can visualize, sketch, or model, just not define.

Most people visualize a point as a tiny, tiny dot. Lines are thought of as long, seamless concatenations of points and planes are composed of finely interwoven lines: smooth, endless and flat.

Think of undefined terms as the basic sounds in a language – the sounds that make up our language for the most part have no meaning in themselves but are combined to make words.

The grammar of our language and a good dictionary are what make the meaning of the sounds. This part of language corresponds to the axioms and definitions that you will find as we move along in the sections and chapters.

From these the facts, flights of fancy, and content-laden sentences are built – these are the theorems and definition in an axiomatic system.

The conventions of the Cartesian plane (17th century, Rene Descartes), the most common visualization of Euclidean Geometry, are well suited to assisting in “seeing” and working with Euclidean geometry. However there are some differences between a geometric approach to points on a line and an algebraic one, as we will see in the explanation of Axiom 3.

SMSG Axioms for Euclidean Geometry:

The axioms are grouped in 5 sections. The first eight axioms deal with points, lines, planes, and distance. Then comes convexity and separation issues – these two axioms deal with facts about the relationships among our undefined objects on a set theoretic basis. Axioms 11 through 14 introduce angles: measuring and constructing them as well as some fundamental facts about linear pairs. With Axiom 15, we begin to look at congruent triangles – note that this is so fundamental a notion that it requires its own axiom. Axiom 16 introduces parallel lines. We then look at area for polygons and congruent triangles (axioms 17 – 20) and we finish up with two axioms about solid figures.

First some words about axioms. Axioms in a system are always true; and they never contradict one another. Axioms discuss the relationships among the undefined terms and they expand what can be done legally in the system. They are the “rules to the game”. They vary greatly from system to system as you will see in video three.

Popper 1.1, Question Two

Which of the following are part of a modern Axiomatic System?

- I. Undefined Terms
 - II. Axioms
 - III. Common notions
 - IV. Theorems
 - V. Definitions
 - VI. Guesses
-
- A. All of these
 - B. III and VI only
 - C. I, II, and III
 - D. I, II, IV, and V
 - E. None of these

MSG Axioms for Euclidean Geometry:

Group 1

A1. Given any two distinct points there is exactly one line that contains them.

Exactly the same as with Euclid.

not



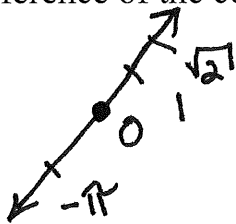
A2. The Distance Postulate:

To every pair of distinct points there corresponds a unique positive number. This number is called the distance between the two points.

A3. The Ruler Postulate:

The points of a line can be placed in a correspondence with the real numbers such that

- A. To every point of the line there corresponds exactly one real number.
- B. To every real number there corresponds exactly one point of the line, and
- C. The distance between two distinct points is the absolute value of the difference of the corresponding real numbers.



$$| -\pi - \sqrt{2} |$$

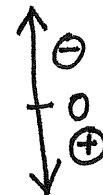
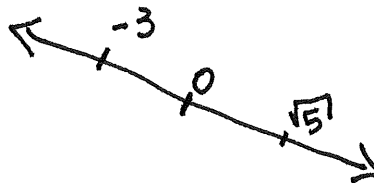
$$| 0 - -\pi |$$

Cartesian

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

A4. The Ruler Placement Postulate:

Given two points P and Q of a line, the coordinate system can be chosen in such a way that the coordinate of P is zero and the coordinate of Q is positive.

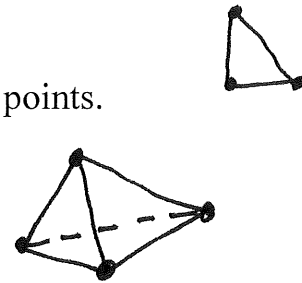


Popper 1.1, Question Three

SMSG Axiom 4 refers only to the standard horizontal x-axis number line.

- A. True
- B. False

- A5. A. Every plane contains at least three non-collinear points.
B. Space contains at least four non-coplanar points.



Popper 1.1, Question Four

“Plane”, “point”, and “space” are undefined terms in this geometry. SMSG Axiom 5 is describing facts and relationships about and among these terms.

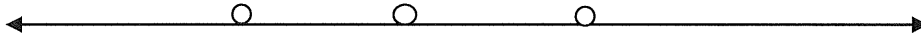
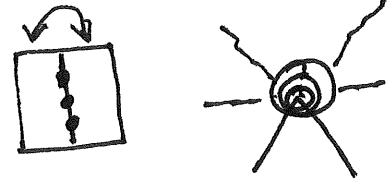
- A. True
- B. False

- A6. If two points line in a plane, then the line containing these points lies in the same plane.



no sagging

A7. Any three points lie in at least one plane, and any three non-collinear points lie in exactly one plane.



A8. If two planes intersect, then that intersection is a line.

Group 2

A9. The Plane Separation Postulate:

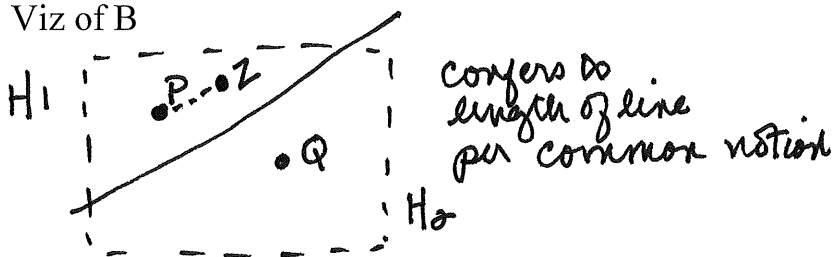
Given a line and a plane containing it, the points of the plane that do not lie on the line form two sets such that

2D

- A. each of the sets is convex, and
- B. if P is in one set and Q is in the other, then segment PQ intersects the line.

Convexity: *given a set, all the points between 2 points in the set are also in the set*

Viz of B



A10. The Space Separation Postulate:

3D

The points of space that do not lie in a given plane form two sets such that

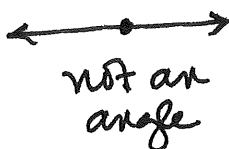
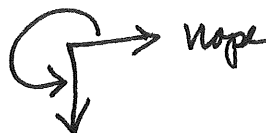
- A. each of the sets is convex, and
- B. if P is in one set and Q is in the other, then the segment PQ intersects the plane.

Group 3

A11. The Angle Measurement Postulate:

To every angle there corresponds a real number between 0° and 180° .

noting! -30° not a thing

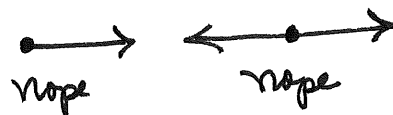


A12. The Angle Construction Postulate:

Let AB be a ray on the edge of the half-plane H . For every r between 0 and 180 there is exactly one ray AP with P in H such that $m \angle PAB = r$.

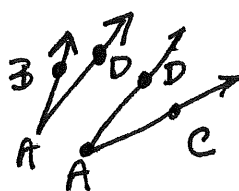
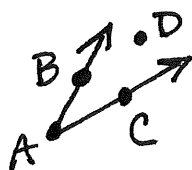


$$0 < r < 180$$



A13. The Angle Addition Postulate:

If D is a point in the interior of $\angle BAC$, then $m \angle BAC = m \angle BAD + m \angle DAC$.



width of AD
= 0
Euclid's "defn"

not a defn just a fact about
our undefined term "line"

A14. The Supplement Postulate:

If two angles form a linear pair, then they are supplementary



not nec 90°/90°

C 90

S 180

"mentary" mnemonic

Group 4

A15. The SAS Postulate:

Given an one-to-one correspondence between two triangles (or between a triangle and itself). If two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.

S_1	A_1	S_{11}	A_{11}
S_2	A_2	S_{12}	A_{12}
S_3	A_3	S_{13}	A_{13}

A16 The Parallel Postulate:

Through a given external point there is at most one line parallel to a given line.



consistent setup across all geometries

Since Euclid's days we have discovered that there are several major categories of geometries. There are geometries that have a finite number of points and no parallel lines, or more than one line parallel to the given line.

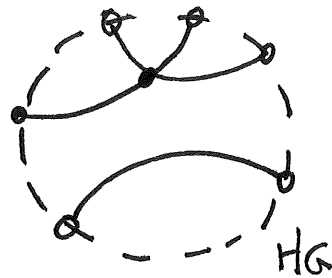
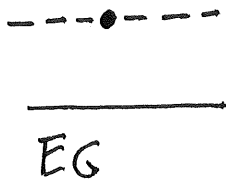
We will look at a couple of these in the next video and as we move through the semester.

There are the Big Three: Euclidean, Spherical and Hyperbolic. Each has it's own axiom system and each is distinctly different about parallel lines. We will pick a line and a point not on that line and see that in

Euclidean: exactly one that shares no points

Spherical: no parallel lines

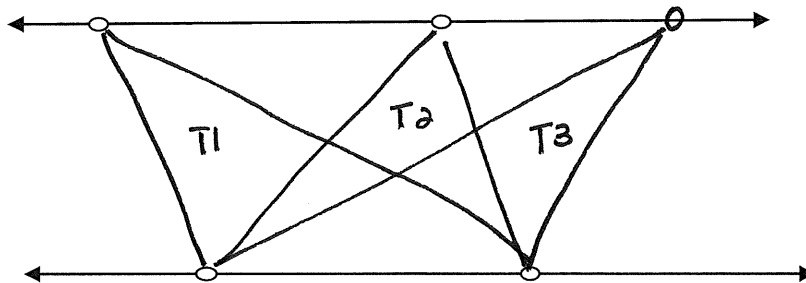
Hyperbolic: more than one line parallel to the given line through the external point.



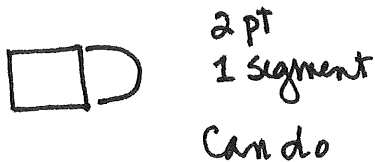
Group 5

A17. To every polygonal region there corresponds a unique positive number called its area.

A18. If two triangles are congruent, then the triangular regions have the same area.



A19. Suppose that the region R is the union of two regions R_1 and R_2 . If R_1 and R_2 intersect at most in a finite number of segments and points, then the area of R is the sum of the areas of R_1 and R_2 .



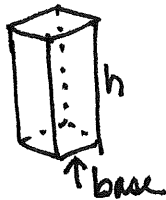
A20. The area of a rectangle is the product of the length of its base and the length of its altitude.

formula



lw $\frac{1}{2}bh$

21. The volume of a rectangular parallelepiped is equal to the product of the length of its altitude and the area of its base.



↑
3D now
↓

A22. Cavalieri's Principal:

Given two solids and a plane. If for every plane that intersects the solids and is parallel to the given plane, the two intersections determine regions that have the same area, then the two solids have the same volume.



Essay 1.1 Number One

What are axioms and why do we care about them?

One page, front side only, 12 point type. PDF and turn in under this title in Assignments in CourseWare.

Popper 1.1, Question Five

Sometimes there are formulas in the axioms.

- A. True
- B. False

This wraps up the modern axiomatic system portion of section 1.1. See you in Video 3! Next up.